Calculus 1   Name: _____

                                   Email address: _____

Maple Lab #2: **Limits, and the Difference Quotient**

(rev. 8/4/15)

Maple is a Computer Algebra System (CAS). In other words, it is a computer program that will perform all kinds of algebra and calculus computations, in addition to all kinds of graphing that can be done on a graphing calculator. The main purpose of this first Calculus I lab is to introduce you to some of the capabilities of the Maple program. (***Notice step 20 on the last page*** *- the lab must be complete and correct before it can be turned in. Please feel free to get help from the lab aide at any time. If you are not doing the lab at the Math Computer Lab at the Academic Success Center, indicate where you did it in the indicated space. You will be contacted if the lab is not complete and correct.)*
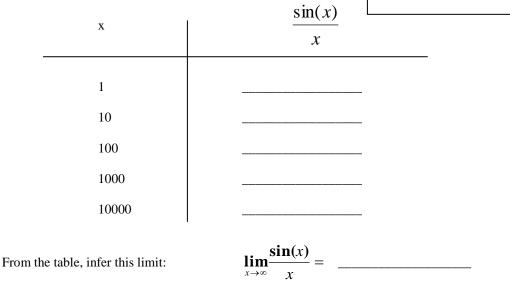
 Part One:

The purpose of this section of lab #2 is to help you understand the concept of the limit of a function, in this case, a trigonometric function.   *(I will not keep telling you about the right arrow.)*

1. Type: **f :=proc(x) evalf(sin(x)/x** *(right arrow)***) end**   You are defining $f(x) = \dfrac{\sin x}{x}$ as a

   function.   It will look like: $f := \mathbf{proc}(x)\,evalf\left(\dfrac{\sin(x)}{x}\right)\mathbf{end}$

2. Type the following commands.
        for k from 0 to 4   do   print(10^k,f(10^k))   od

> *These commands tell Maple to plug the numbers 1, 10, etc. up to 100,000 into the function $f(x) = \dfrac{\sin x}{x}$ and outputs the results.*

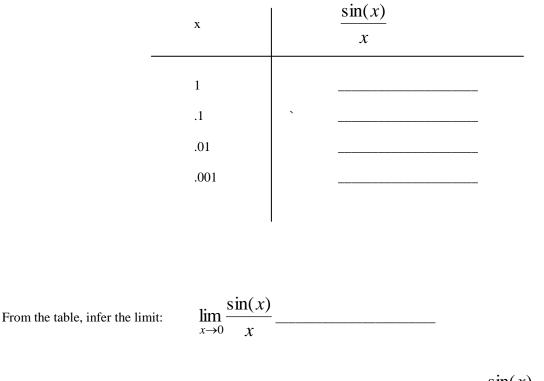Fill in the following table:

x	$\dfrac{\sin(x)}{x}$
1	_____
10	_____
100	_____
1000	_____
10000	_____

From the table, infer this limit: $\displaystyle\lim_{x\to\infty}\dfrac{\sin(x)}{x} =$ _____

3. Now type: (or cut and paste)

For   k from 0 to 4   do   print(.1^k, f(.1^k))   od

Fill in the following table:

x	$\dfrac{\sin(x)}{x}$
1	_____
.1	_____
.01	_____
.001	_____

From the table, infer the limit: $\displaystyle\lim_{x\to 0} \dfrac{\sin(x)}{x}$ _____

3. Use the information that you have found to sketch a rough graph of $f(x) = \dfrac{\sin(x)}{x}$ by hand.

4.   Now, plot the graph of $f(x) = \dfrac{\sin(x)}{x}$ by typing: **plot(f(x), x = -50..50);** Does the graph of the function   corroborate what you determined to be the limit as x goes to zero? Explain.

How about as the limit of $f(x) = \dfrac{\sin(x)}{x}$ as x goes to infinity? Explain how the graph

leads to the same conclusion as the table?

_____

_____

Part Two:

The derivative of the function f at a is defined by

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Even though there are easier ways to calculate most derivatives, it is essential to understand this formula for a couple of reasons. First, all of the theorems and formulas that you come across will use this definition. Second, understanding this definition will help in your understanding of the meaning of the derivative – specifically **DERIVATIVE MEANS SLOPE!!!!**

We will divide the application of the definition of the derivative into several steps.

1. First, you will define the function for which you want to find the derivative. This function is $f(x) = x^2 - 5x$.

   Type: **f := proc(x) x^2 – 5 * x   end**

2. Next, you will find the y value of this function at a point that is h units to the right of

   Type: **f(1+h);**   What does Maple return?

   _____

3. In this next step, you are finding the difference between the function value at 1 + h, and the function value at 1.

   Type: **(f(1 + h) – f(1))**   Maple returns with:

   _____

4. Now type: **(f(1 + h) – f(1))/h**
              simplify(%)   What does maple return? _____

Explain in words what this expression represents. _____

_____

5. Now find the limit of that expression as h goes to 0:
   Type: **limit(%, h=0)**

What do you get? _____ What does that value represent?_____

_____

6. You should have ended up with an answer of –3, which is in fact the value of $f'(x)$ when x = 1. By replacing all occurrences of the number 1 by the letter x in the previous sequence of steps, you can determine the derivative of this function at the unspecified value "x". You can go back and make the changes in the lines already typed, or type the following commands:

f(x + h)

f(x + h) – f(1)

(f(x + h) – f(x))/h

simplify(%)

limit(%, h=0)

What is your final result? _____

7. Maple has a built-in command that determines the derivative of f at a given point, as well as the derivative at an unspecified point (x, f(x)). To verify the previous results, type

   diff (f(x) , x)
   subs(x = 1, %) Did you get the same results? _____

8. The next few steps will get you an equation of the line tangent to f(x) at x = 1. First, define the derivative of f(x) as a function.

   Type: **fprime := proc(x) 2x – 5 end;**
   Now it is easier to find the derivative when x = 1.

   Type: **fprime(1).** Remember that you can find the function value by typing **f(1)**

9. These functions can be used to find the equation of a line tangent to the graph of f(x) at any specified value of x using the point – slope form for the equation of a line $y - y_1 = m(x - x_1)$.

Type: **tanlineat1 := y – f(1) = fprime(1)*(x – 1)**

10. If you solve this equation for y, you can graph it together with the graph of f(x).

Type: **tanlineat1 := solve(tanlineat1, y)**
plot({f(x), tanlineat1}, x = -2 .. 4)

Sketch the graph:

11. This lab is complete and correct.

_____   _____
lab aide   date

I completed this lab on my own at   or

_____   _____
                                                                                    date

12. How long did it take you to complete this lab? _____

| | Strongly agree | strongly disagree |
13. Doing this lab gave me a better understanding of derivatives. 5 4 3 2 1

14. Comments . . .