Revised 3/19/15

Calculus II Name: ____________________

Email: ____________________

Maple Lab #3: Integration By Parts

The main purpose of this Maple lab is to give you some insight into and practice with the method of integration by parts, and a type of formula that can be derived from it, called a reduction formula. *(Notice step 4 on the last page . When you do the lab at the Learning and Advising Center and get the signature of the lab assistant, you are guaranteed credit for it. If you are not doing the lab at the Math Computer Lab at the Learning and Advising Center, indicate where you did it in the indicated space. You will be contacted if the lab is not complete and correct.)*

Integration by parts is the integration version of the product rule.

$$\int f(x)g(x)dx = f(x)G(x) - \int f'(x)G(x)dx$$

However, the alternative form is probably more familiar:

$$\int udv = uv - \int vdu$$

(Everything in bold is what you type.)

1. The integral that we will be working with first is

$$\int xsin(x)dx$$

Working with Maple allows you go quickly check a couple of different choices for u and dv, to see which yields the simplest integral for $\int vdu$

with(student)

First, we identify $\int vdu$ as z.

z := Int(x * sin(x),x)

This next statement tells Maple to do the integration by parts method, using sin(x) as the u value.

intparts(z, sin(x)) That implies dv = $\boxed{}$

The result is

Unfortunately, this was a poor choice for "u" because the new integral is more complicated than the original one. So, for the next time we define u as "x", which makes dv =

intparts(z,x)

This time the result is

The integral portion of the result is easy now, and the final answer is

2. Now we will work with the integral

$$\int e^x sinx dx$$

z:= Int(exp(x)*sin(x),x)

First, we choose e^x as u. That implies dv =

intparts(z, exp(x))

The result is

$\int e^x sinx dx =$

Well, that's not any worse,, . . . but it is not any better either. Let's see what happens if we choose sin (x) as u. That would make dv = this time.

intparts(z, sin(x))

Hmmm, that's almost the same as the first try. What could that mean? Can it be that this integration can be done either way? Let's go back to the result we got when we let $u = e^x$

$$\int e^x sinxdx = -\, e^x \cos(x) + \int e^x cos(x)dx$$

Let's integrate the $\int e^x cos(x)dx$ by parts this time.

vdu := Int(exp(x)*cos(x),x)

intparts(vdu, exp(x))

$$\int e^x sinxdx = -\, e^x \cos(x) + \left(\overbrace{\boxed{\text{Result goes here}}}^{\int e^x cox(x)dx} \right)$$

But wait . . . . .isn't the integral part the same integral we have on the left side of the equation? Suppose you add $\int e^x \sin(x)\, dx$ to both sides of the equation? Then multiply both sides by one-half?

$$\int e^x sinxdx =$$

Now, on your own, showing all work, find

$\int e^x sinxdx$ by letting sin(x) be u.

$\int e^x sinxdx =$

You should get the same result!

3. When an integral contains an expression that is raised to a high power, it is sometimes possible to use integration by parts to come up with a formula to rewrite the integral utilizing a lower power. This is known as a reduction formula. Suppose you wanted to find a general formula for

$$\int x^n e^x dx$$

where n is a positive integer. Again, we define the integral:

z := Int(x^n*exp(x), x)

and then decide to make $u = x^n$, implying that $dv = e^x dx$. (Why is this a better choice than $u = e^x, dv = x^n dx?$)

intparts(z, x^n)

simplify (%)
The result is $\int x^n e^x dx =$

Do you notice a pattern? So from this formula, $\int x^{n-1} e^x dx$ should then be:

$x^n e^x - n \Big[$ Result goes here $\Big]$

If you distribute the n, you get

$$x^n e^x - n x^{n-1} e^x - n(n-1)\int x^{n-2} e^x dx$$

As you can see, the second term is the same as the first term, except that its first factor is the derivative of the first factor of the previous term. If you were to keep going, every addition term would have the same pattern ((derivative of previous first factor)*(e^x)).

Where will it all end? Well, we would finally get down to an integrable integral when ***n*** gets down to 0:

$$\int x^0 e^x dx = \int e^x dx = e^x + C$$

So we would say according to the reduction formula,

$$\int x^n e^x dx \quad = \quad x^n e^x - n x^{n-1} e^x + n(n-1)e^x \quad \dots \dots . + (-1)^n n!\, e^x + C$$

Use the reduction formula to find

$$\int x^4 e^x dx$$

$\int x^4 e^x dx =$

Check your answer on Maple: int(x^4*exp(x),x)

Was your answer correct? What is the problem with Maple's answer?

4. a. This lab is complete and correct.

_________________________ _______________ OR:

lab aide date

b. I didn't do the lab at the Learning and Advising Center.

I did it at:

____

How long did it take you to complete this lab? ______________________________

	Strongly agree				strongly disagree
Doing this lab gave me a better understanding of integration.	5	4	3	2	1